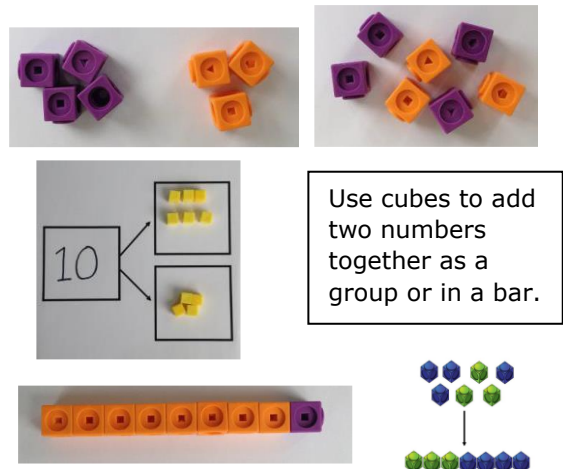
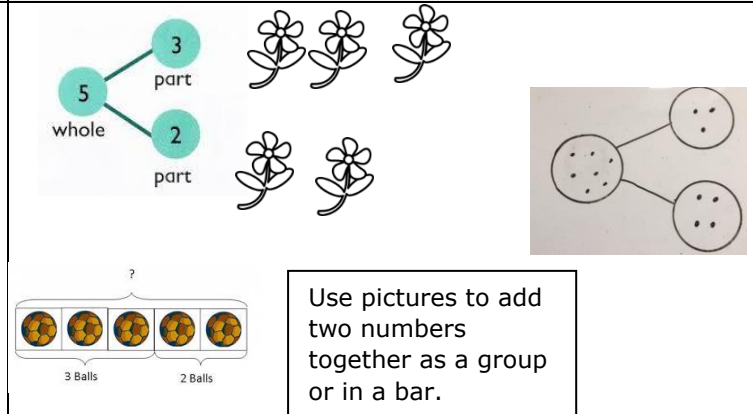
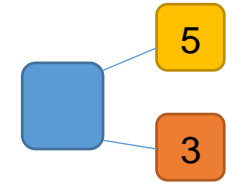
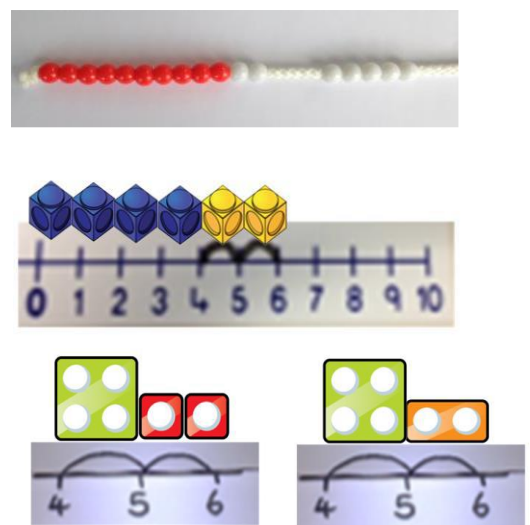
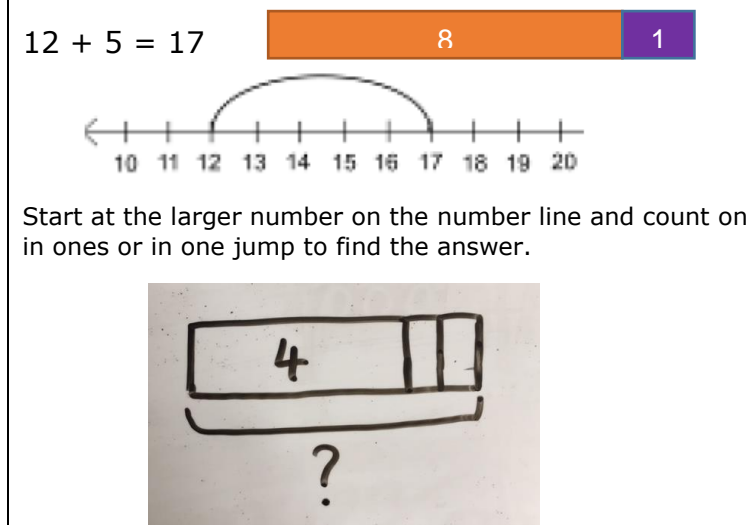


Progression in Calculations


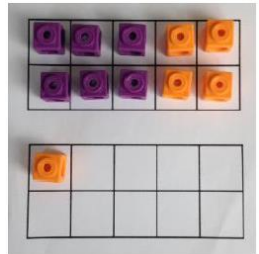
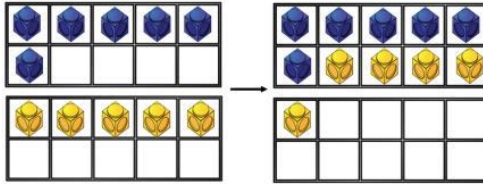
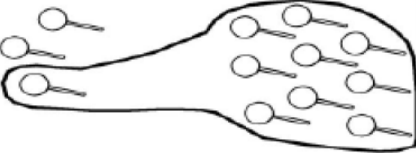
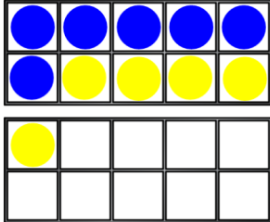
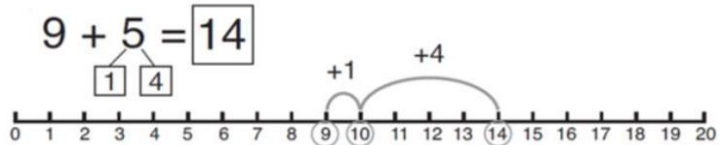

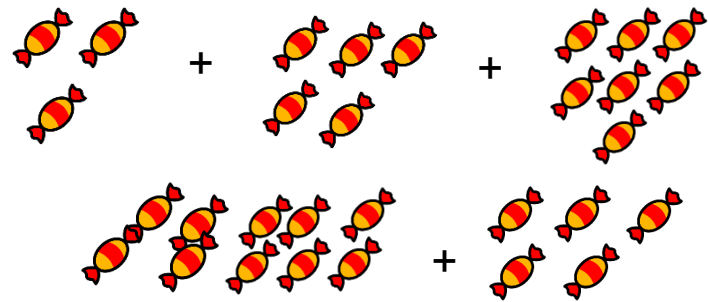
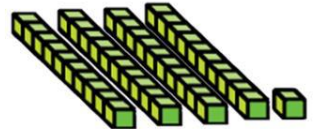
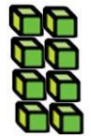
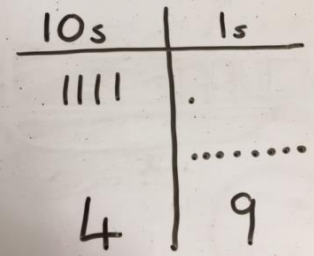
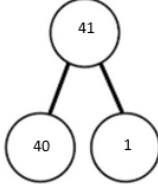
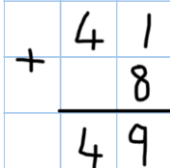
	EYFS	YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5	YEAR 6
ADDITION	<p>Combining two parts to make a whole: part whole model.</p> <p>Starting at the bigger number and counting on - using cubes.</p> <p>Regrouping to make 10 using ten frame.</p>	<p>More written recordings</p> <p>Combining two parts to make a whole: part whole model.</p> <p>Starting at the bigger number and counting on - using cubes.</p> <p>Regrouping to make 10 using ten frame.</p>	<p>Adding three single digits.</p> <p>Use of base 10 to combine two numbers.</p> <p>Column method - regrouping.</p>	<p>Column method - regrouping. (up to 3 digits using place value counters)</p>	<p>Column method - regrouping. (up to 4 digits)</p>	<p>Column method - regrouping.</p> <p>Use of place value counters for adding decimals.</p>	<p>Column method - regrouping.</p> <p>Abstract methods.</p> <p>Place value counters to be used for adding decimal numbers.</p>
SUBTRACTION	<p>Taking away ones</p> <p>Counting back</p> <p>Find the difference</p> <p>Part whole model</p> <p>Make 10 using the ten frame</p>	<p>More written recordings</p> <p>Taking away ones</p> <p>Counting back</p> <p>Find the difference</p> <p>Part whole model</p> <p>Make 10 using the ten frame</p>	<p>Counting back</p> <p>Find the difference</p> <p>Part whole model</p> <p>Make 10</p> <p>Use of base 10</p> <p>Column method with regrouping.</p>	<p>Column method with regrouping. (up to 3 digits using place value counters)</p>	<p>Column method with regrouping. (up to 4 digits)</p>	<p>Column method with regrouping.</p> <p>Abstract for whole numbers.</p> <p>Start with place value counters for decimals - with the same amount of decimal places.</p>	<p>Column method with regrouping.</p> <p>Abstract methods.</p> <p>Place value counters for decimals - with different amounts of decimal places.</p>
MULTIPLICATION	<p>Recognising and making equal groups.</p> <p>Doubling</p> <p>Counting in multiples</p> <p>Use cubes, Numicon and other objects in the classroom</p>	<p>More written recordings</p> <p>Recognising and making equal groups.</p> <p>Doubling</p> <p>Counting in multiples</p> <p>Use cubes, Numicon and other objects in the classroom</p>	<p>Arrays - showing commutative multiplication</p>	<p>Arrays</p> <p>2d x 1d using base 10</p>	<p>Column multiplication - introduced with place value counters. (2 and 3 digit multiplied by 1 digit)</p>	<p>Column multiplication</p> <p>Abstract only but might need a repeat of year 4 first (up to 4 digit numbers multiplied by 1 or 2 digits)</p>	<p>Column multiplication</p> <p>Abstract methods (multi-digit up to 4 digits by a 2 digit number)</p>
DIVISION	<p>Sharing objects into groups</p> <p>Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups?</p> <p>Use cubes and draw round 3 cubes at a time.</p>	<p>More written recordings</p> <p>Sharing objects into groups</p> <p>Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups?</p> <p>Use cubes and draw round 3 cubes at a time.</p>	<p>Division as grouping</p> <p>Division within arrays - linking to multiplication</p> <p>Repeated subtraction</p>	<p>Division with a remainder - using lollipop sticks, times tables facts and repeated subtraction.</p> <p>2d divided by 1d using base 10 or place value counters</p>	<p>Division with a remainder</p> <p>Short division (up to 3 digits by 1 digit - concrete and pictorial)</p>	<p>Short division (up to 4 digits by a 1 digit number including remainders)</p>	<p>Short division</p> <p>Long division with place value counters (up to 4 digits by a 2 digit number)</p> <p>Children should exchange into the tenths and hundredths column too</p>

Progression in Calculations

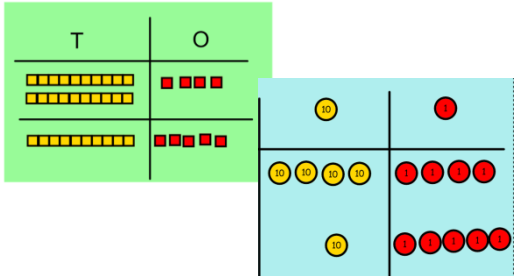
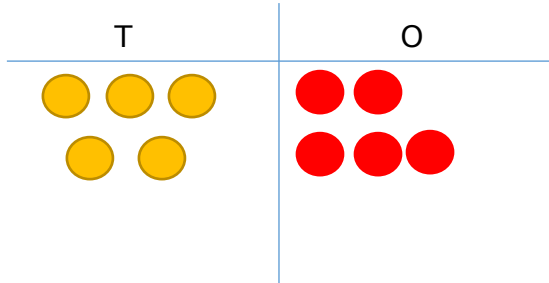
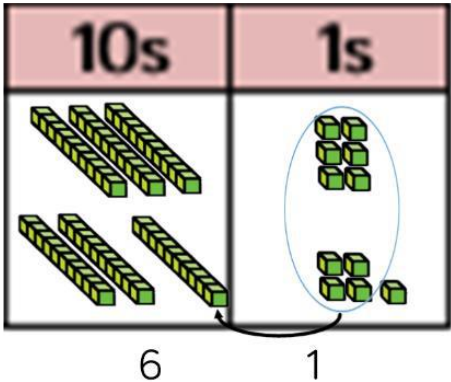
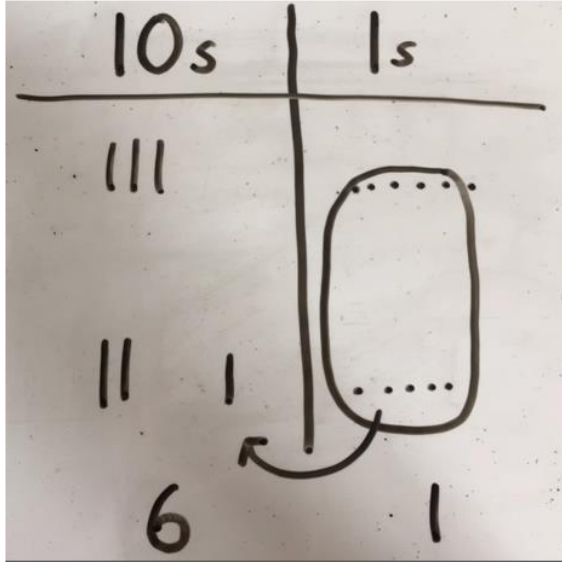
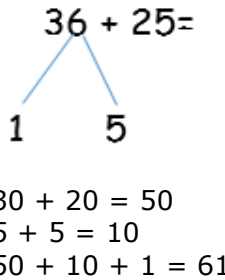
Addition

Strategies	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part-whole model</p>	 <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$</p> <p>$10 = 6 + 4$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>
<p>Starting at the bigger number and counting on</p>		<p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p> <p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2 =$</p>

Progression in Calculations

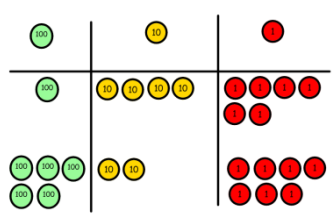
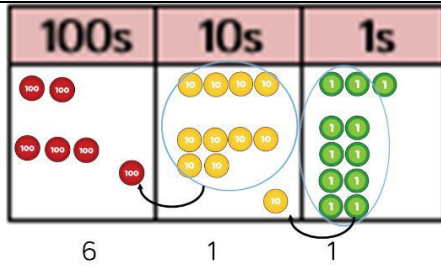
<p>Regrouping to make 10.</p>	 <p style="text-align: center;">$6 + 5 = 11$</p> <p>Start with the bigger number and use the smaller number to make 10.</p>  	 <p>Use pictures or a number line. Regroup or partition the smaller number to make 10.</p> <p style="text-align: center;">$3 + 9 =$</p>  <p style="text-align: center;">$9 + 5 = 14$</p> 	<p>$7 + 4 = 11$</p> <p>If I am at seven, how many more do I need to make 10? How many more do I add on now?</p> <p>$6 + \square = 11$</p> <p>$6 + 5 = 5 + \square$</p> <p>$6 + 5 = \square + 4$</p>
<p>Adding three single digits</p>	<p>$4 + 7 + 6 = 17$ Put 4 and 6 together to make 10. Add on 7.</p>  <p>Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.</p>	<p>Add together three groups of objects. Draw a picture to recombine the groups to make 10.</p> 	<p>$4 + 7 + 6 = 10 + 7$</p> <p style="text-align: center;">10</p> <p style="text-align: center;">$= 17$</p> <p>Combine the two numbers that make 10 and then add on the remainder.</p>
<p>TO + O</p>	<p>Continue to develop understanding of partitioning and place value. $41 + 8$</p>  	<p>Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.</p> 	<p>$41 + 8$ $1 + 8 = 9$ $40 + 9 = 49$</p>  

Progression in Calculations

<p>Column method - no regrouping</p>	<p>$24 + 15 =$ Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.</p> 	<p>After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.</p> 	<div style="border: 1px dashed black; padding: 5px;"> <p style="text-align: center;"><u>Calculations</u></p> <p>$21 + 42 =$</p> $\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$ </div> <p style="text-align: right;">Formal method</p>
<p>Column method - regrouping TO + TO</p>	<p>Continue to develop understanding of partitioning and place value.</p> <p>$36 + 25 =$</p> 	<p>Children to represent the base 10 in a place value chart.</p> 	<p>Looking for ways to make 10.</p> <p>$36 + 25 =$</p>  <p style="text-align: right;">Formal method</p> $\begin{array}{r} 36 \\ + 25 \\ \hline 61 \\ \hline 1 \end{array}$
<p>Column method - regrouping</p>	<p>Make both numbers on a place value grid.</p>	<p>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.</p>	<p>Start by partitioning the numbers before moving on to clearly show the exchange below the addition.</p>

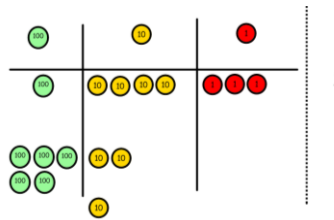
Progression in Calculations

Larger numbers



146
+ 527

Add up the units and exchange 10 ones for one 10.

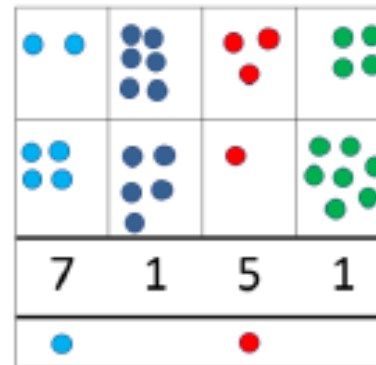
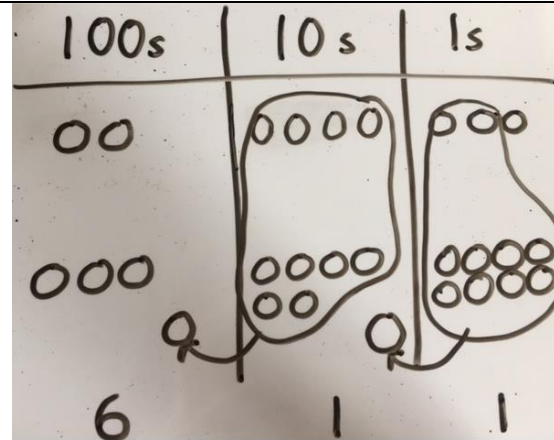


146
+ 527

Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.



$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

$$\begin{array}{r} 25 \\ + 48 \\ 13 \\ \hline 60 \\ \hline 73 \end{array}$$

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array}$$

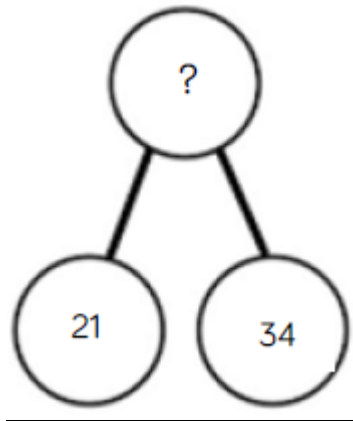
$$\begin{array}{r} £ 23.59 \\ + £ 7.55 \\ \hline £ 31.14 \\ 111 \end{array}$$

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ 212 \end{array}$$

Progression in Calculations

Addition – Conceptual Variation

Different ways to solve $21 + 34$



?	
21	34

Word problems:

In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

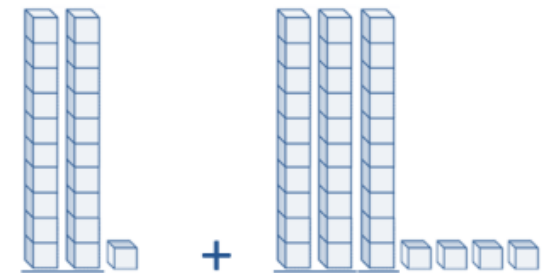
$21 + 34 = 55$. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \hline \end{array}$$

$21 + 34 =$

= $21 + 34$

Calculate the sum of twenty-one and thirty-four.

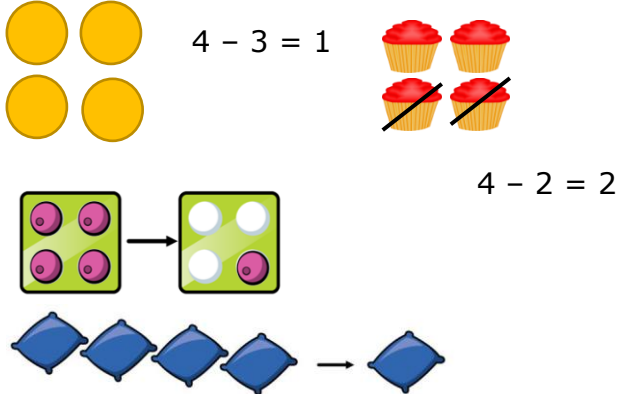
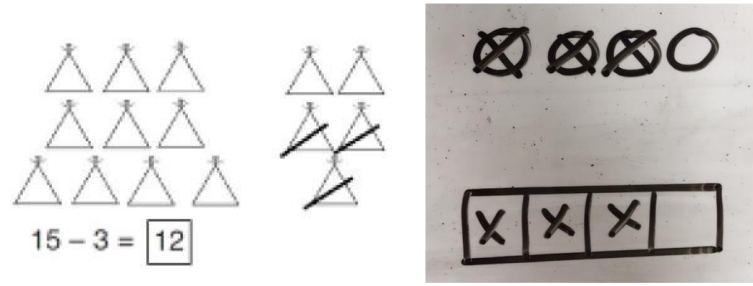
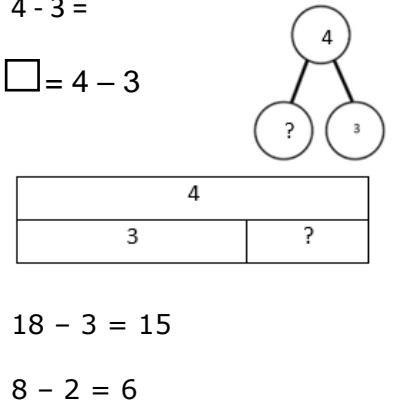
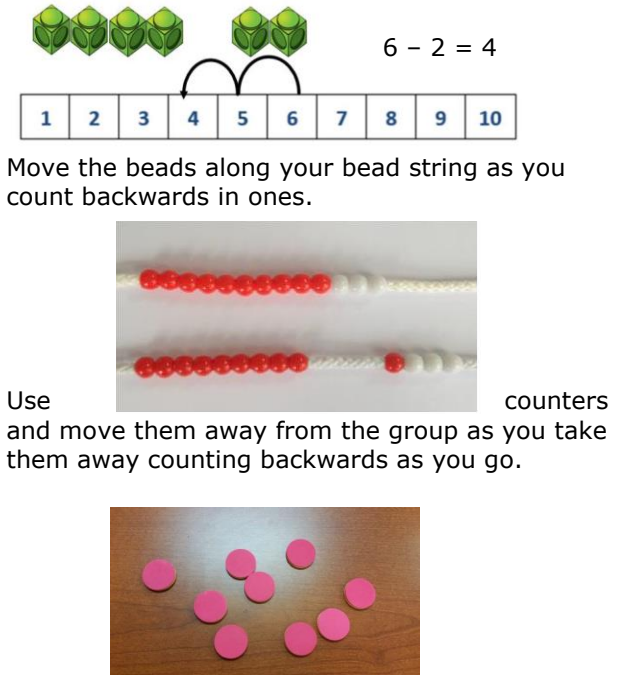
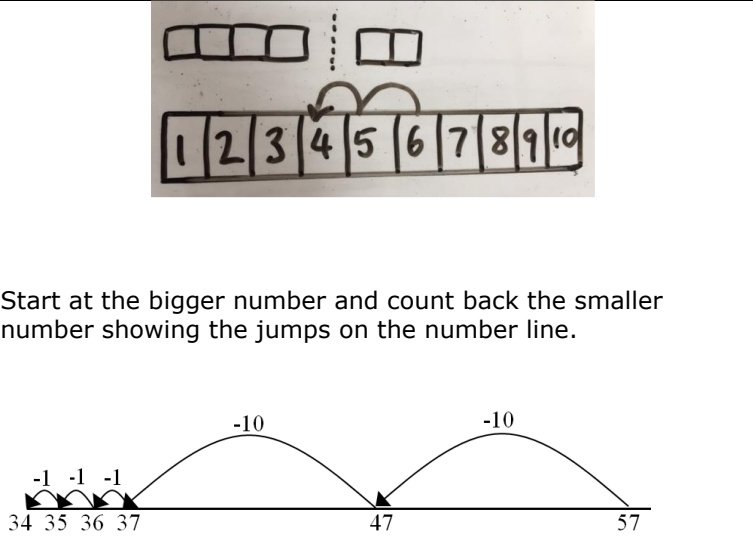
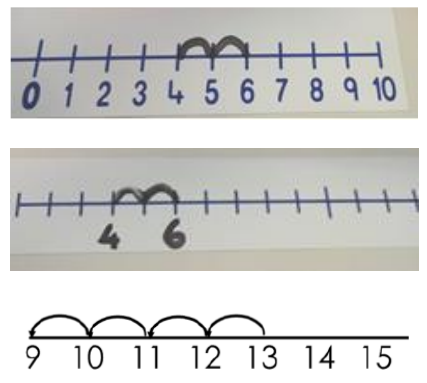


Missing digit problems:

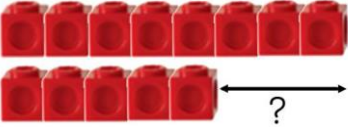
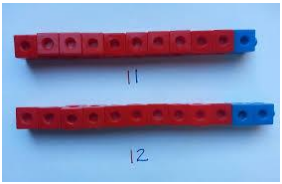
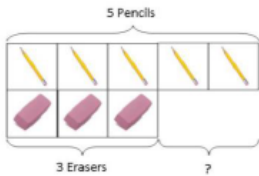
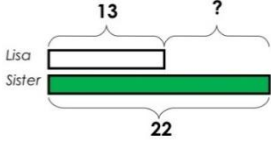
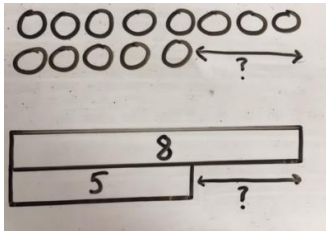
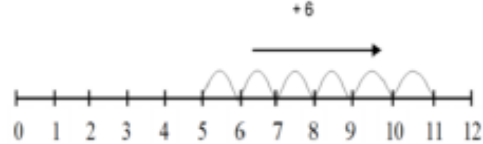
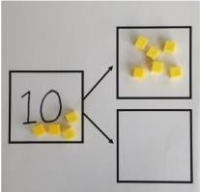
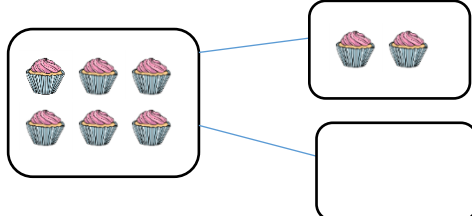
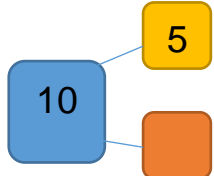

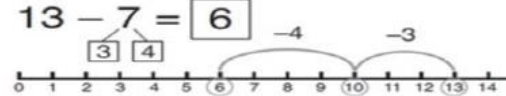

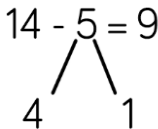
10s	1s
	?
?	5

Progression in Calculations

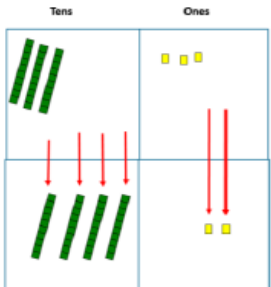
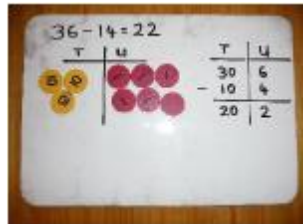
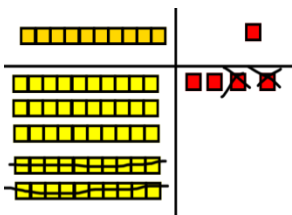
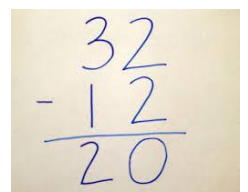
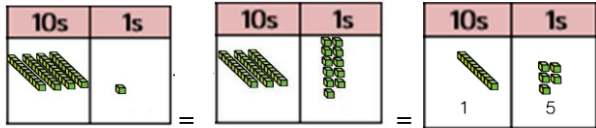
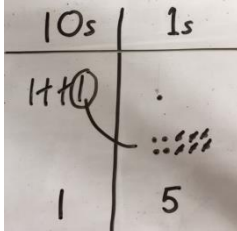
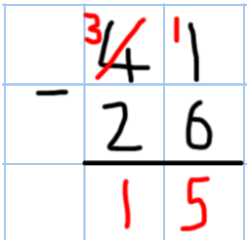
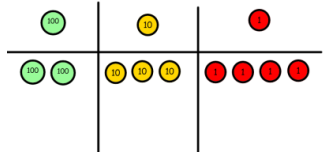
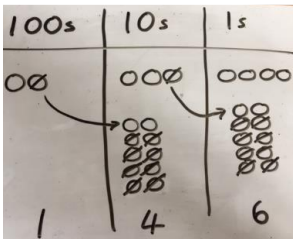
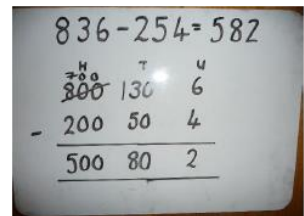
Subtraction

Strategies	Concrete	Pictorial	Abstract
<p>Taking away ones</p>	<p>Use physical objects, counters, cubes etc. to show how objects can be taken away.</p>  <p>$4 - 3 = 1$</p> <p>$4 - 2 = 2$</p>	<p>Cross out drawn objects to show what has been taken away.</p>  <p>$15 - 3 = 12$</p>	<p>$4 - 3 =$</p> <p>$\square = 4 - 3$</p>  <p>$4 - 3 = 1$</p> <p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p>
<p>Counting back</p>	 <p>$6 - 2 = 4$</p> <p>Move the beads along your bead string as you count backwards in ones.</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p>	 <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p> <p>$34 \quad 35 \quad 36 \quad 37 \quad \quad \quad 47 \quad \quad \quad 57$</p> <p>$-1 \quad -1 \quad -1$</p> <p>$-10 \quad -10$</p> <p>This can progress all the way to counting back using two 2 digit numbers.</p>	<p>Put 13 in your head, count back 4. What number are you at?</p> <p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.</p> 

Progression in Calculations

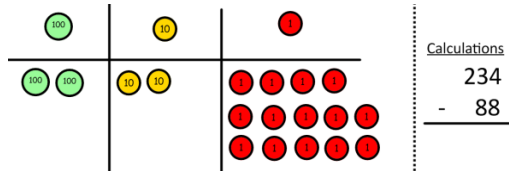
<p>Find the difference</p>	<p>Compare amounts and objects to find the difference. Calculate the difference between 8 and 5.</p>  <p>Use cubes to build towers or make bars to find the difference</p>   <p>Use basic bar models with items to find the difference</p>	<p>Draw bars to find the difference between 2 numbers.</p> <p style="text-align: center;">Comparison Bar Models</p> <p><i>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</i></p>   <p>Count on to find the difference.</p> 	<p>Find the difference between 8 and 5.</p> <p>8 - 5, the difference is ?</p> <p>Children to explore why $9 - 6 = 8 - 5 = 7 - 4$ all have the same difference.</p> <p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p>
<p>Part Part Whole Model</p>	<p>Link to addition - use the part whole model to help explain the inverse between addition and subtraction.</p>  <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> <p style="text-align: center;">$10 - 6 =$</p>	<p>Use a pictorial representation of objects to show the part part whole model.</p> 	 <p>Move to using numbers within the part whole model.</p>
<p>Make 10</p>	<p>$14 - 9 =$</p>  <p>Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.</p>	<p>$13 - 7 = 6$</p>  <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p> 	<p>$16 - 8 =$</p> <p>How many do we take off to reach the next 10? How many do we have left to take off?</p> <p>$14 - 5 = 9$</p>  <p style="text-align: right;">$14 - 4 = 10$ $10 - 1 = 9$</p>

Progression in Calculations

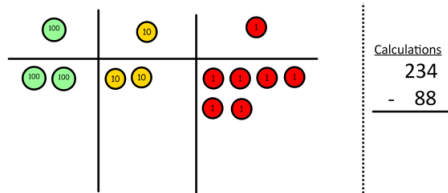
<p>Column method without regrouping</p>	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;">  <p style="text-align: center;">$75 - 42 = 33$</p> <p>Use Base 10 to make the bigger number then take the smaller number away.</p> </div> <div style="width: 45%;"> <p>Show how you partition numbers to subtract. Again make the larger number first.</p>  </div> </div>	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;">  <p style="text-align: center;">Calculations</p> $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$ </div> <div style="width: 45%;"> <p>Draw the Base 10 or place value counters alongside the written calculation to help to show working.</p> </div> </div>	<p>This will lead to a clear written column subtraction.</p> $47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ 
<p>Column method with regrouping TO - TO</p>	<p>Column method using base 10 and having to exchange. $41 - 26$</p> 	<p>Represent the base 10 pictorially remembering to show the exchange.</p> 	<p>Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.</p> 
<p>Column method with regrouping</p> <p>Larger numbers</p>	<p>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.</p> <p>Make the larger number with the place value counters</p>  <p style="text-align: right;">Calculations</p> $\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$	<p>Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.</p> 	<p>Children can start their formal written method by partitioning the number into clear place value columns.</p> 

Progression in Calculations

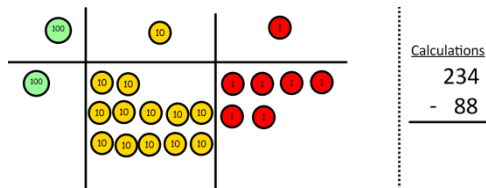
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



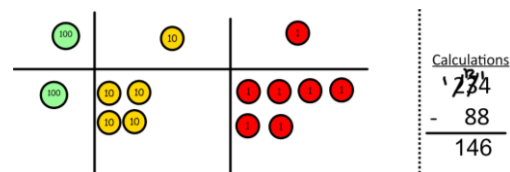
Now I can subtract my ones.



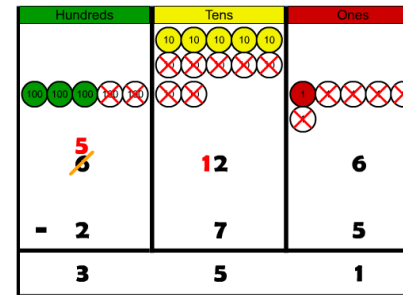
Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Now I can take away eight tens and complete my subtraction

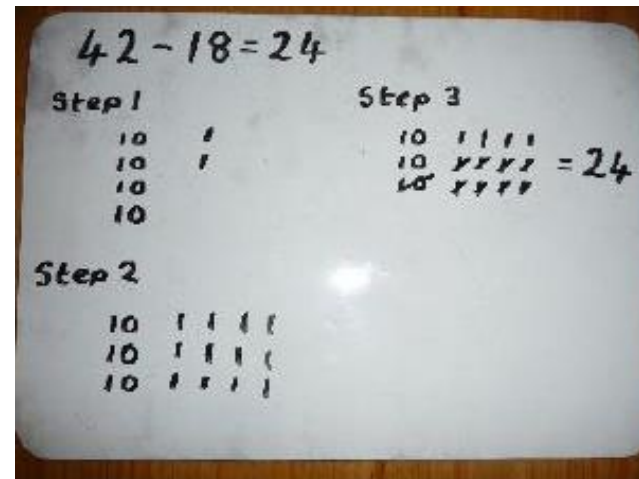


Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.



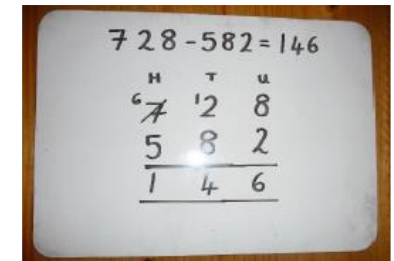
When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.



Moving forward the children use a more compact method.

$$\begin{array}{r} ^2 ^1 \\ 234 \\ - 88 \\ \hline 146 \end{array}$$



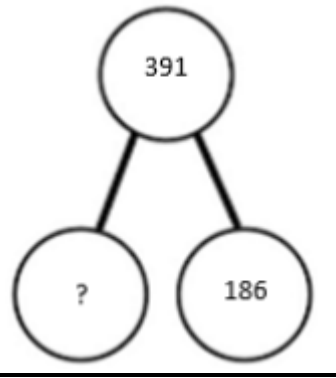
This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} ^5 ^{12} ^1 \\ 236.0 \\ - 26.5 \\ \hline 236.5 \end{array}$$

Progression in Calculations

Subtraction – Conceptual Variation

Different ways to solve $391 - 186$



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

= $391 - 186$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

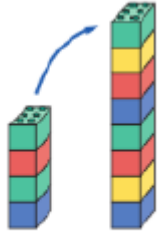

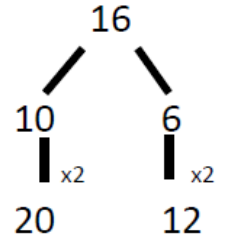
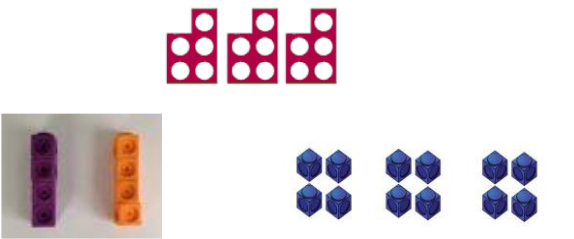
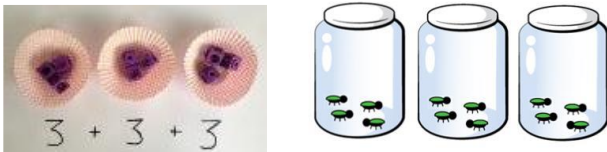
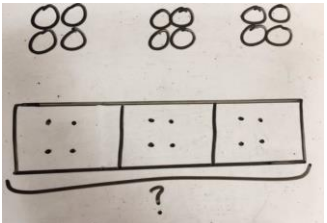

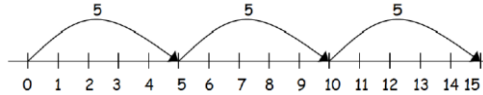

What is 186 less than 391?

Missing digit calculations



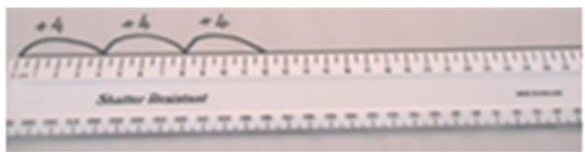
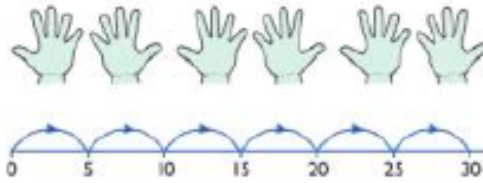
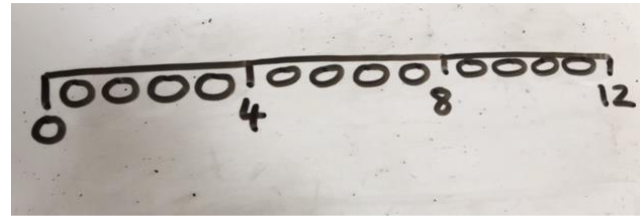
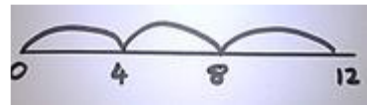

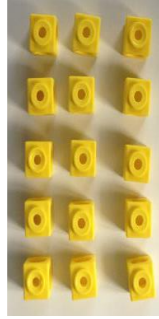
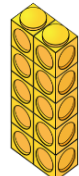
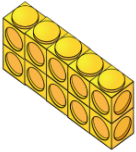

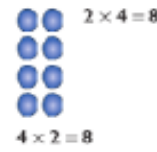
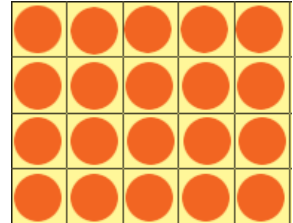
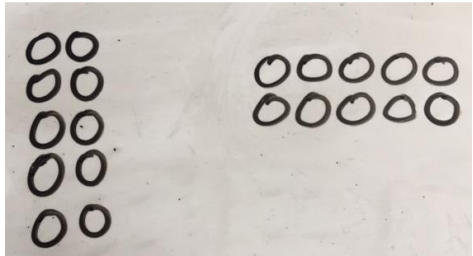

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

Progression in Calculations

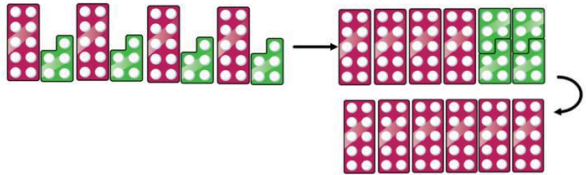
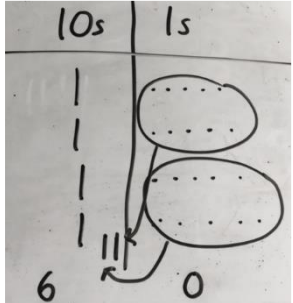
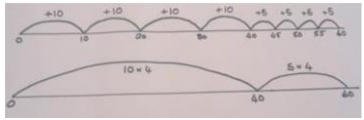
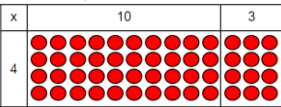
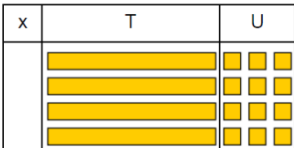
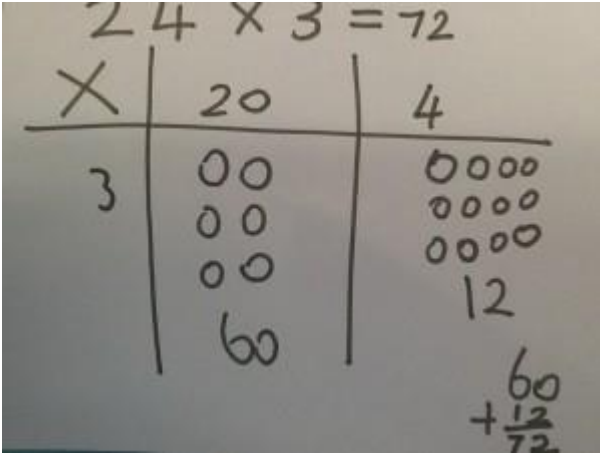
Multiplication

Strategies	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities to show how to double a number.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double a number.</p> <p style="text-align: center;">Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
Repeated addition	<p>Use different objects to add equal groups.</p>   <p>$3 + 3 + 3$</p>	<p>Children to represent the practical resources in a picture and use a bar model.</p>  <p>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?</p>  <p>2 add 2 add 2 equals 6</p>  <p>$5 + 5 + 5 = 15$</p>	<p>Write addition sentences to describe objects and pictures.</p>  <p>$2 + 2 + 2 + 2 + 2 = 10$</p> <p>$3 \times 4 = 12$ $4 + 4 + 4 = 12$</p>

Progression in Calculations

<p>Counting in multiples</p> <p>Number lines</p>	<p>Count in multiples supported by concrete objects in equal groups.</p>   	<p>Children to represent the practical resources in a picture and use a bar model.</p>  	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p> <p>Abstract number line showing three jumps of four.</p> $3 \times 4 = 12$ 
<p>Arrays - showing commutative multiplication</p>	<p>Create arrays using counters/ cubes to show multiplication sentences.</p>    <p>2 lots of 5</p>  <p>5 lots of 2</p>	<p>Draw arrays in different rotations to find commutative multiplication sentences.</p>    <p>Link arrays to area of rectangles.</p> 	<p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p>$5 + 5 + 5 = 15$</p> <p>$3 + 3 + 3 + 3 + 3 = 15$</p> <p>$5 \times 3 = 15$</p> <p>$3 \times 5 = 15$</p>

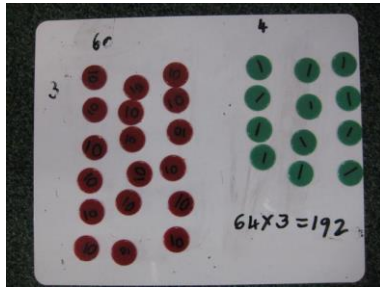
Progression in Calculations

<p>Partition to multiply</p>	<p>Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4×15</p> 	<p>Children to represent the concrete manipulatives pictorially.</p> 	<p>Children to be encouraged to show the steps they have taken.</p> $4 \times 15 \quad \begin{array}{l} 10 \times 4 = 40 \\ 5 \times 4 = 20 \\ 40 + 20 = 60 \end{array}$ 																														
<p>Grid Method</p>	<p>Show the link with arrays to first introduce the grid method.</p> <p>4 rows of 10 4 rows of 3</p>   <p>Move on to using Base 10 to move towards a more compact method. 4 rows of 13</p>	<p>Children can represent the work they have done with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">30</td> <td style="padding: 5px;">5</td> </tr> <tr> <td style="padding: 5px;">7</td> <td style="padding: 5px;">210</td> <td style="padding: 5px;">35</td> </tr> </table> <p style="text-align: center;">$210 + 35 = 245$</p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td></td> <td style="text-align: center;">10</td> <td style="text-align: center;">8</td> </tr> <tr> <td style="text-align: center;">10</td> <td style="text-align: center;">100</td> <td style="text-align: center;">80</td> </tr> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">30</td> <td style="text-align: center;">24</td> </tr> </table> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">1000</td> <td style="padding: 5px;">300</td> <td style="padding: 5px;">40</td> <td style="padding: 5px;">2</td> </tr> <tr> <td style="padding: 5px;">10</td> <td style="padding: 5px;">10000</td> <td style="padding: 5px;">3000</td> <td style="padding: 5px;">400</td> <td style="padding: 5px;">20</td> </tr> <tr> <td style="padding: 5px;">8</td> <td style="padding: 5px;">8000</td> <td style="padding: 5px;">2400</td> <td style="padding: 5px;">320</td> <td style="padding: 5px;">16</td> </tr> </table>	x	30	5	7	210	35		10	8	10	100	80	3	30	24	x	1000	300	40	2	10	10000	3000	400	20	8	8000	2400	320	16
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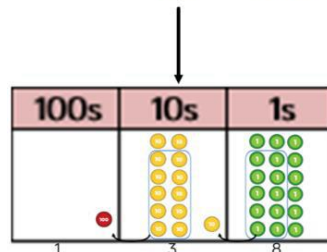
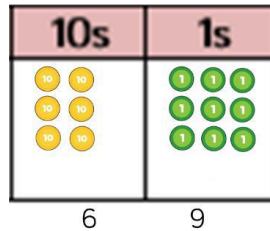
Progression in Calculations

Column multiplication

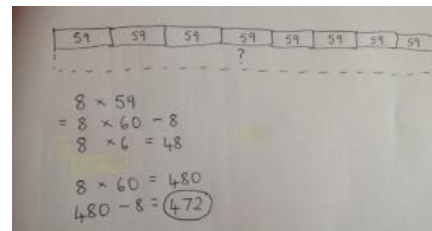
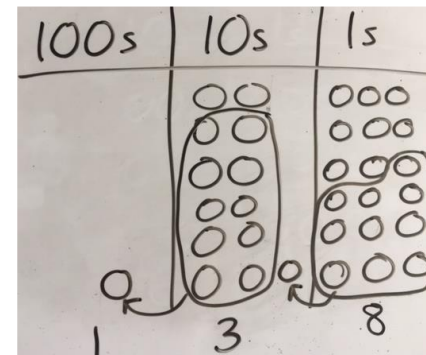
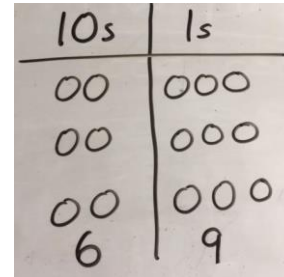
Children can continue to be supported by place value counters at the stage of multiplication.



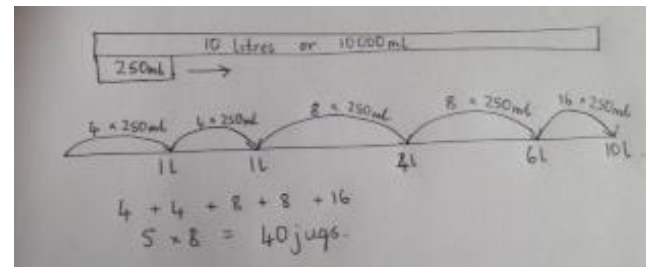
It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.



Children to represent the counters pictorially.



Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



Children to record what it is they are doing to show understanding.

$$3 \times 23 \quad 3 \times 20 = \bar{6}0$$

$$\begin{array}{r} 20 \\ 3 \end{array} \quad \begin{array}{r} 3 \times 3 = 9 \\ 60 + 9 = 69 \end{array}$$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

$$\begin{array}{r} 32 \\ \times 24 \\ \hline 8 \quad (4 \times 2) \\ 120 \quad (4 \times 30) \\ 40 \quad (20 \times 2) \\ 600 \quad (20 \times 30) \\ \hline 768 \end{array}$$

If it helps, children can write out what they are solving next to their answer.

$$\begin{array}{r} 7 \quad 4 \\ \times 6 \quad 3 \\ \hline 1 \quad 2 \\ 2 \quad 1 \quad 0 \\ 2 \quad 4 \quad 0 \\ 4 \quad 2 \quad 0 \quad 0 \\ \hline 4 \quad 6 \quad 6 \quad 2 \end{array}$$

$$\begin{array}{r} 1 \quad 2 \quad 4 \\ \times 2 \quad 6 \\ \hline 7 \quad 4 \quad 4 \\ 1 \quad 2 \\ 2 \quad 4 \quad 8 \quad 0 \\ \hline 3 \quad 2 \quad 2 \quad 4 \\ 1 \quad 1 \end{array}$$

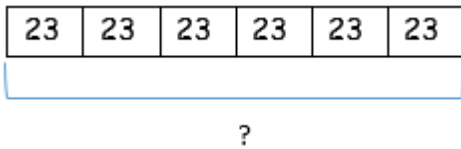
This moves to the more compact method.

Answer: 3224

Progression in Calculations

Multiplication – Conceptual Variation

Different ways to solve 6×23



Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

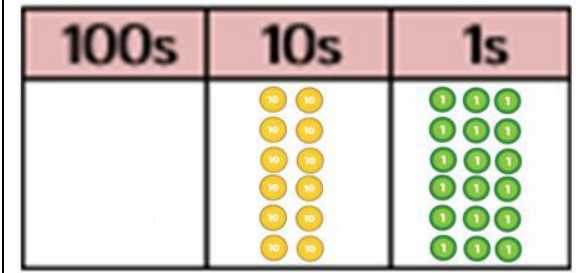
Find the product of 6 and 23.

$6 \times 23 =$

= 6×23

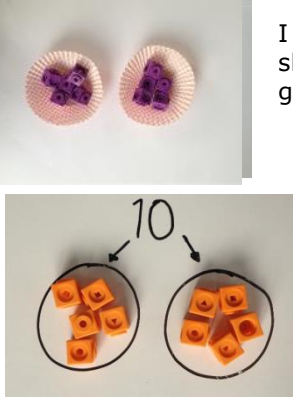
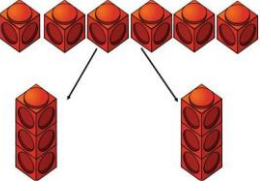
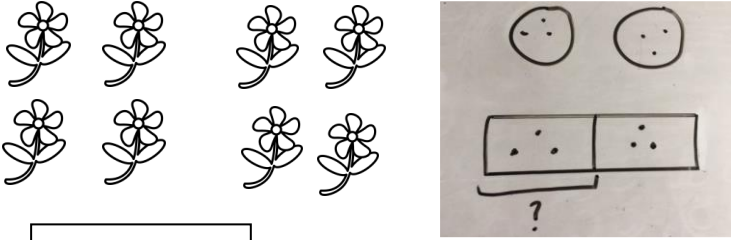
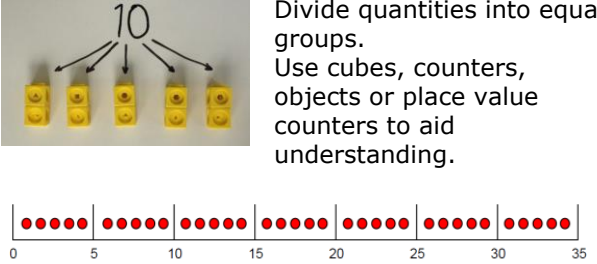
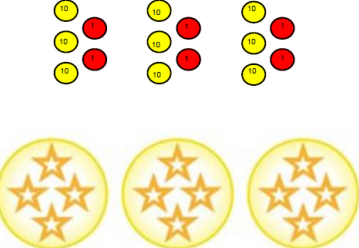
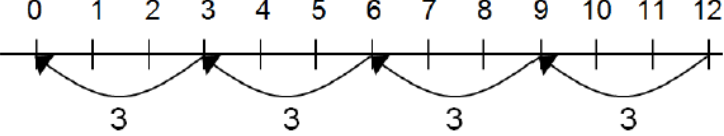
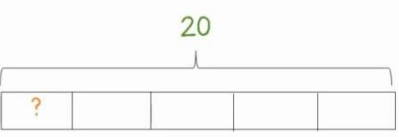
$$\begin{array}{r}
 6 \\
 \times 23 \\
 \hline
 23 \\
 \hline
 \times 6 \\
 \hline
 \hline
 \hline
 \end{array}$$

What is the calculation?
What is the product?



Progression in Calculations

Division

Strategies	Concrete	Pictorial	Abstract
<p>Sharing objects into groups</p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p> 	<p>Children use pictures or shapes to share quantities.</p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $8 \div 2 = 4$ </div>	<p>Share 9 buns between three people.</p> $9 \div 3 = 3$
<p>Division as grouping</p>	 <p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p> $96 \div 3 = 32$ 	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	$28 \div 7 = 4$ <p>Divide 28 into 7 groups. How many are in each group?</p>

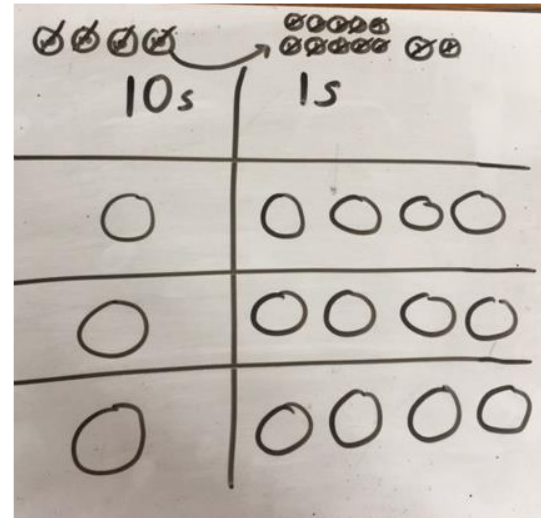
Progression in Calculations

Sharing using place value counters

$42 \div 3 = 14$

The diagram illustrates the process of dividing 42 by 3 using place value counters. It shows two stages: first, 4 tens and 2 ones are represented in a grid; second, after exchanging 1 ten for 10 ones, there are 3 tens and 12 ones, which are then shared into 14 groups of 3 ones each.

Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

$$42 \div 3$$

$$42 = 30 + 12$$

$$30 \div 3 = 10$$

$$12 \div 3 = 4$$

$$10 + 4 = 14$$

Short division

Use place value counters to divide using the bus stop method alongside.

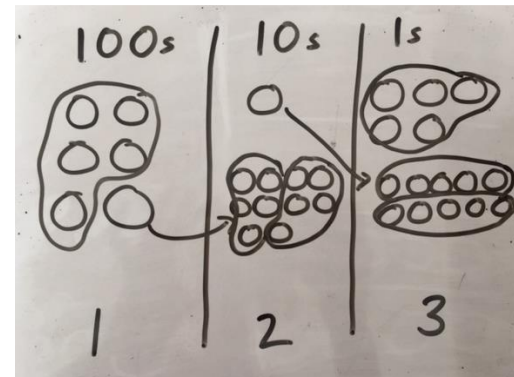
$615 \div 5$

The diagram shows 615 represented with place value counters: 6 hundreds (red), 1 ten (yellow), and 5 ones (green). A vertical line separates the hundreds, tens, and ones. The hundreds are grouped into 3 groups of 2, the ten into 3 groups of 1, and the ones into 3 groups of 1. The final result is 3 groups of 205.

1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.

Encourage them to move towards counting in multiples to divide more efficiently.



Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 3 \overline{) 4872} \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$$

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$$

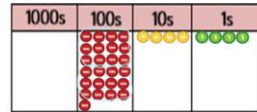
Progression in Calculations

Long division

Long division using place value counters
 $2544 \div 12$

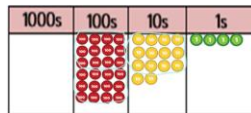


We can't group 2 thousands into groups of 12 so will exchange them.



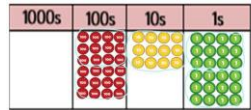
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

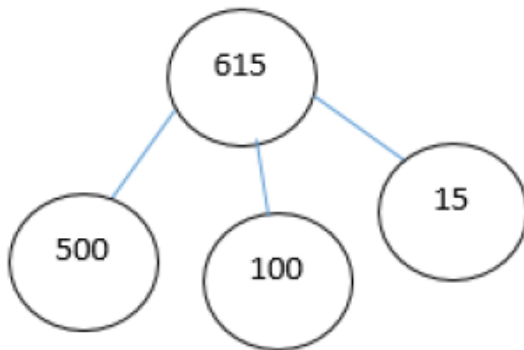
$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Progression in Calculations

Division – Conceptual Variation

Different ways to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$615 \div 5 =$

= $615 \div 5$

$$\begin{array}{r} 5 \overline{) 615} \end{array}$$

What is the calculation?
What is the answer?

100s	10s	1s